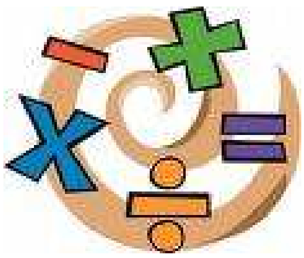
MATHS

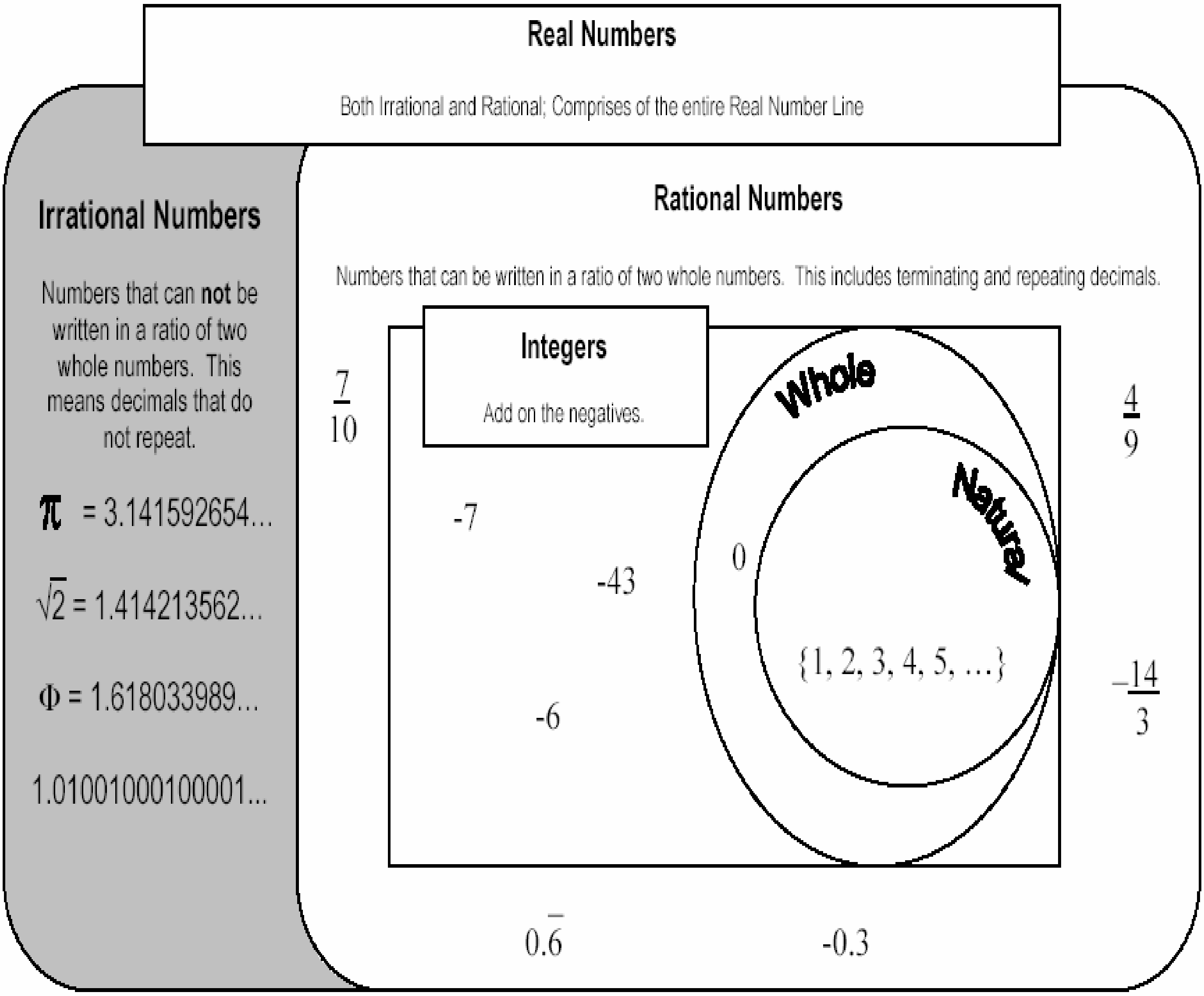
for 3rd of E.S.O

I.E.S. LLANES



NUMBERS

CLASSIFYING NUMBERS



REAL NUMBERS

1. Find these amounts:
   - **4/5 of 35 km = ...**
   - **¼ of 240 ml = ...**
   - **2 ¼ times 450 grams = ...**
2. What fraction of the second amount is the first?
   - **25 cm, 75 cm**
   - **630 kg, 1.5 tonnes**
3. Change decimal into fractions and vice versa:

   1/9, 0.22222..., 0.6030303......, 5/6

4. Classify these numbers as in natural numbers N, integers Z, rational numbers Q, and irrational numbers

   I: 3.75555.., 1/8, -5, π, $\sqrt{16}$**, 0.010010001......**

5. Round:
   - $\sqrt{3}$ to the hundredths
   - π to the thousandth
6. Write, if it is possible:
   - *An irrational number less than 3 and bigger than 2.*
   - *A negative rational and non integer number between -11 and -10*
   - *An integer and irrational number*
7. Solve these questions:
   - *In a class of 32 pupils 1/8 are left-handed. How many are not left-handed?*
   - *Colin earns £ 25 500 a year. This year he has a 3% pay rise. How much does Colin now earns?*
   - *A coat costs £ 140. In a sale it's rediced to £ 85. What is the percentage reduction?*
   - *The cost for a ticket has risen by 15% to £ 23. What was the original price?*
   - *The price for a CVD player has been reduced by 20% in a sale. It now costs £ 180. What was the original price?*
8. Write this list of numbers in order of size. Start with the smallest.

   25%, 1/3, 0.35, 2/5, 0.333..., 72%, 2/7, π/5

POWERS & ROOTS

LAWS OF INDICES

- $a^m \cdot a^n = a^{m+n}$
- $a^m : a^n = a^{m-n}$
- $(a^m)^n = a^{m \cdot n}$
- $a^0 = 1$
- $a^1 = a$
- $a^{-n} = 1/ a^n$
- $a^{m/n} = \sqrt[n]{a^m}$
- $a^n \cdot b^n = (a \cdot b)^n$
- $a^n : b^n = (a:b)^n$

STANDARD INDEX FORM

Standard form (or **scientific notation**) is a way of writing very large or very small numbers, by expressing them, as the product of a power of 10 and a number that is greater than or equal to 1 and less than 10. It is also known as **exponential form.**

Examples:

$600\ 000\ 000 = 6 \cdot 10^8$

$0.000\ 000\ 00015 = 1.5 \cdot 10^{-10}$

$32\ 400 = 3.24 \cdot 10^4$

$0.000\ 047\ 56 = 4.756 \cdot 10^{-5}$

Surds or radicals

If you can't simplify a number to remove a square root (or cube root etc), then it is a surd.

Example: $\sqrt{25} = 5$ is a natural number

$\sqrt{2}$ can't be simplified, so it's a surd.

Number	Simplifed	As a Decimal	Surd or not?
$\sqrt{2}$	$\sqrt{2}$	1.4142135(etc)	**Surd**
$\sqrt{3}$	$\sqrt{3}$	1.7320508(etc)	**Surd**
$\sqrt{4}$	2	2	*Not a surd*
$\sqrt{(1/4)}$	1/2	0.5	*Not a surd*
$\sqrt[3]{(27)}$	3	3	*Not a surd*

As you can see, the surds have a decimal which goes on forever without repeating, and that makes them Irrational Numbers.

LAWS OF SURDS

Remember! $\sqrt[n]{a^m} = a^{m/n}$

Simplifying: $\sqrt[n \cdot p]{a^{m \cdot p}} = \sqrt[n]{a^m}$

Multiplication:

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{(a \cdot b)}$

Division:

$\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{(a:b)}$

9. Simplify these expressions:

$a)\ 2^4 \cdot 5^4 \div 10^3$ $b)\ (2^4)^3 \cdot 2^{-5} \cdot 2^{-3}$ $c)\ (2^2 \cdot 2)^{-2}$ $d)\ ((-3)^{-2})^{-3}$

$e)\ (5x)^2$ $f)\ (2x^2y^3)^{-4}$ $g)\ (2x^{-3})^3$

10. Evaluate the following, giving your answers as fractions:

$a)\ 2^{-5} =$ .... $b)\ \left(\frac{5}{7}\right)^{-2} =$ .... $c)\ 8^{\frac{1}{3}} =$ .... $d)\ 4^{-\frac{3}{2}} =$ ....

11. Decide wheather these numbers are written in standard form:

$a)\ 249000 = 24.9 \cdot 10^4$ $b)\ 0.047 = 47 \cdot 10^{-3}$ $c)\ 0.000\ 0096 = 9.6 \cdot 10^{-6}$

12. Work out these calculations. Give your answers in standard form:

$a)\ (2 \cdot 10^7) \cdot (9 \cdot 10^{-4}) =$ ....

$b)\ (6.3 \cdot 10^{-3}) \cdot (2 \cdot 10^6) =$ ....

$c)\ (9 \cdot 10^{12}) \div (3 \cdot 10^{-4}) =$ ....

$d)\ (2.4 \cdot 10^{10}) \div (3 \cdot 10^6) =$ ....

13. The mass of an atom is $2 \cdot 10^{-23}$ grams. What is the total mass of $9 \cdot 10^{15}$ of these atoms?

14. If $3.8 \cdot 10^8$ seeds weigh 1 kilogram and each seed weighs the same, calculate the weight in grams of one seed. Give your answer in standard form, correct to 3 significant figures.

15. Which expression is equivalent to $\sqrt{12}$?

- $2\sqrt{6}$ • $2\sqrt{3}$ • $3\sqrt{2}$ • $6\sqrt{2}$

16. Remove factors from these radicals:

- $\sqrt{24} =$ ....
- $\sqrt{75} =$ ....
- $\sqrt{48} =$ ....
- $\sqrt{12} =$ ....

17. Simplify these:

- $\sqrt[4]{7^6} =$ .... • $\sqrt[6]{5^{15}} =$ .... • $\sqrt[15]{1000} =$ .... • $\sqrt[6]{64}$

18. Change these surds into powers and do the operations:

- $\sqrt{2} \cdot \sqrt[3]{2} =$ ....
- $\sqrt[3]{2} \cdot \sqrt[4]{4} =$ ....
- $\sqrt[6]{7} \div \sqrt[4]{7} =$ ....
- $\sqrt[3]{5} \cdot \sqrt{5} \div \sqrt[4]{5^3} =$ ....

PROPORTIONALITY

- **Ratio** is a way of comparing amounts of something. It shows how much bigger one thing is than another. For example: Use 3 parts blue paint to 1 part white.

- **Proportion** is an equation with a ratio on each side. It is a statement that two ratios are equal. Parts of a proportion:

- **Direct proportion**: *If two quantities are directly proportional then their quotient is an invariant(it does not change, it is constant).* Example in the table:

A	2	3	4	5
B	6	9	12	15

- **Inverse proportion**: *If two quantities are inversly proportional, then their product is an invariant (it does not change, it is constant).* Example in the table:

C	12	6	3	2
D	2	4	8	12

19. A ship that is 120 m long is modelled on a scale of 1:250. How long is the model?

20. If 4/7 of a tank can be filled in 2 minutes, how many minutes will it take to fill the whole tank?

21. It takes 4 men 6 hours to repair a road. How long will it take 7 men to do the job if they work at the same rate?

22. A car travels 125 miles in 3 hours. How far would it travel in 5 hours?

23. In an army camp, there is food for 8 weeks for 1200 people. After 3 weeks, 300 more soldiers joined the camp. For how many more weeks will the food last?

24. 24 people can construct a house in 15 days. But the owner would like to finish the work in 12 days. How many more workers should he employ?

25. Vicky and Tracy share £ 14 400 in the ratio 4:5. Work out how much each of them receives.

26. Mrs London inherited £ 55 000. She divided the money between her children in the ratio 3:3:5. How much did each child receive?

27. The price of a television including VAT *(IVA)* at 17.5% is £ 235. Work out the cost of the television before VAT was added.

28. Charlotte has £ 4250 in the bank. If the interest rate is 6.8% p.a. *(per annum)*, how much interest on the saving will she get at the end of the year?

29. A car costs £ 6000 cash, or can be bought by hire purchase *(compra a plazos)* with a 30% deposit followed by 12 monthly instalments of £ 365. Find:

    a. The deposit

    b. The total amount paid for the car on hire purchase.

30. A flat was bought in 1998 for £ 62 000. The price increased by 20% in 1999 and then by a further 35% in 2000. How much was the flat worth at the end of 2000? *(to be worth= valer)*

31. There are 12500 aliens in a spaceship. $\frac{5}{20}$ of them are from Grogoom planet, $\frac{2}{7}$ are from Sylvan- Darktowers and the remainder are from Darth-Moon. How many aliens of each kind are there in the spaceship?

32. Bearwoods forest has many ancient trees. $\frac{2}{3}$ of them are oaktrees, $\frac{1}{4}$ of the remainder are pineapples. If the final remainder are 6000000 trees, how many trees are there in Bearwoods forest?

33. Vicky dropped a ball and it bounced to a height of $\frac{2}{5}$ of the height Vicky dropped it. It bounced again and reached $\frac{2}{5}$ of the previous height and so on. After 3 bounces the ball reached 0,32m. What height did Vicky drop the ball?

ALGEBRA

Algebra is a simple language, it is the use of symbols (letters) to represent numbers.

Since algebra uses the same symbols as arithmetic for adding, subtracting, multiplying and dividing, you're already familiar with the basic vocabulary.

An ***algebraic expression*** is one or more algebraic terms in a phrase. It can include variables, constants, and operating symbols.

In Algebra "Substitution" or "Evaluation"means putting numbers where the letters are and working out the result: *If x=5 and y=3, then what is **10/x + 2y** ?*
Put "5" where "x" is, and "3" where "y" is: **10/5 + 2×3 = 2 + 6 = 8**

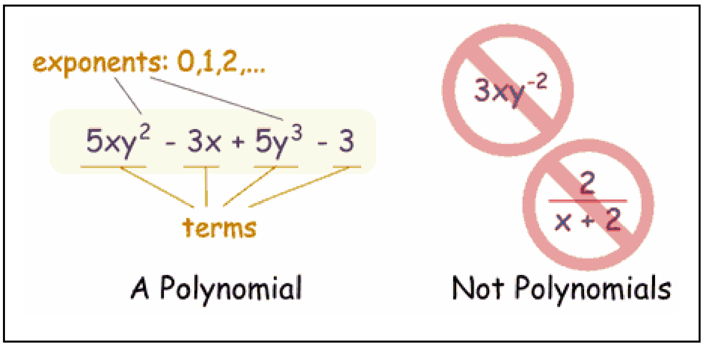
Polynomial: It can be made of:

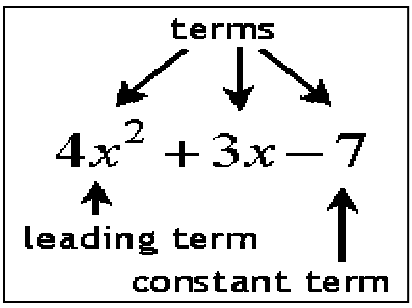
- ✓ **constants** (like **3**, **-20**, or $\frac{1}{2}$)
- ✓ **variables** (like ***x*** and ***y***)
- ✓ **exponents** (like the 2 in y^2) but they can only be **0, 1, 2, 3, ...**

That can be combined using:

+ - × addition, subtraction and multiplication, ...

✗ **... but not division!** ✗





OPERATIONS WITH POLYNOMIALS

Monomials that have the same variable factors are called **like terms.** For example, $4x^2$ and $5x^2$ are like terms.

Add like terms by adding the numbers
in front of the terms, following the rules for adding signed numbers. Example: $4x^2 + 5x^2 = (4 + 5)\,x^2 = 9x^2$

Subtract like terms by changing the signs of the terms being subtracted, and following the rules for adding polynomials. Example: $^2 - 4) - (x^2 + 3x - 3) = 2x^2 - x^2 - 3x - 4 + 3 = \mathbf{x^2 - 3x - 1}$

Multiply polynomials by using the distributive property: $(a+b) \cdot (c+d) = ac + ad + bc + bd$; then, combine like terms.

Special binomial patterns or remarkable identities:

Squaring a Binomial $\quad \mathbf{(a + b)^2 = a^2 + 2ab + b^2}$
$\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$

Product of Sum and Difference $\quad \mathbf{(a+b)(a-b) = a^2 - b^2}$

EXERCISES ABOUT POLYNOMIALS

1. Answer these questions:
   a. What is the leading term of the polynomial $2x^9 - 4xy^7 + 121$?
   b. Give an example of a polynomial of degree five with exactly 3 terms.
   c. How many terms does a complete polynomial with one variable of degree 4 have?
2. Evaluate the following:
   d. $2x^3 - x^2 - 4x + 2$ at $x = 1/2$
   e. $2 - x^2 - x^3$ at $x = -1$
3. Simplify:
   f. $x + 2(x - [3x - 8] + 3)$
   g. $[(6x - 8) - 2x] - [(12x - 7) - (4x - 5)]$
   h. $(x + 3)(x + 2)$
   i. $(4x^2 - 4x - 7)(x + 3)$
   j. $(3x^2 - 9x + 5)(2x^2 + 4x - 7)$
   k. $(4x^4 + 3x^3 + 2x + 1):(x^2 + x + 2)$
   l. $(2x^3 - 9x^2 + 15):(2x - 5)$
   m. $(3x^3 - 5x^2 + 10x - 3):(3x + 1)$
4. Expand these remarkable identities:
   a. $(2x-3)^2 - (2x+1)\cdot(x-3) =$ ....
   b. $\left(\frac{1}{2} + 3x^2\right)^2 =$ ....
   c. $(5a+2)(5a-2) - (5a+2)\cdot(a-1) =$ ....
   d. $(2x+1)^2 - (2x-1)^2 =$ ....

5. Factorise:

$$a.\ 4x^3 + 8x^2 + 2x = ...$$

$$b.\ 36y^2 - 9y - 12 = ...$$

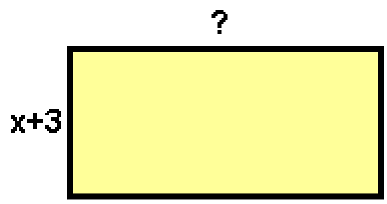
$$c.\ a^2 - 6a + 9 = ...$$

$$d.\ 16b^2 - 4b^3 = ...$$

$$e.\ \frac{3}{4}x^2 - \frac{9}{2}x - \frac{3}{4} = ...$$

6. Translate these sentences into algebraic language:
   - *n. 4 more than 3 times t.*
   - *o. 4 less than half of y.*
   - *p. 6 more than a, divided by n.*
   - q. *c less than a quarter of p.*
   - *r. The perimeter and the area of a rectangle with a height 3 cm less than the base.*
   - *s. The product of three consecutive whole numbers.*
   - *t. The square of the sum of two numbers.*
   - ***u.*** *The sum of the square of two numbers.*

7. The room that is shown in the figure below has a floor space of $2x^2 + x - 15$ square feet. If the width of the room is (x + 3) feet, what is the length?



EQUATIONS

- An **equation** is a mathematical statement asserting the equality of two expressions that involves unknowns. Finding the **solution or root** is called **solving** the equation.

  3a-2 = a+1
- Always perform **the same** operation on both sides of an equation.
- To "undo" the effect of an operation, apply the inverse (opposite) operation.
- Solving linear equations:
  - 1st remove brackets
  - 2nd remove denominators
  - Transpose terms and colect like terms
  - Get the solution
- Solving factorised equations: set each factor equal to zero.
- Solving quadratic equations $ax^2 + bx + c = 0$
  - Incomplete equations with b = 0: calculate x^2, then get x.
  - Incomplete equations with c = 0: factor the equation (x is common factor).
  - Complete equation: use the quadratic formula

    $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- The **discriminant** is the name given to the expression that appears under the square root (radical) sign in the quadratic formula.
- The **sum** of the roots of a quadratic equation is equal to the negation of the coefficient of the second term divided by the leading coefficient.

  $$x_1 + x_2 = -b/a$$
- The **product** of the roots of a quadratic equation is equal to the constant term divided by the leading coefficient.

  $$x_1 \cdot x_2 = c/a$$

SIMULTANEOUS EQUATIONS

Simultaneous Equations

Simultaneous equations are a set of equations which have more than one value which has to be found

There are two methods of solving simultaneous equations:

Substitution:

1. *Isolate one of the variables ('x') on one side of one of the equations*:
2. *Substitute for the isolated variable in the other equation:*

We will then have one equation in one unknown, which we can solve

Elimination or addition:

1. *Make one pair of coefficients negatives of one another,*
2. *Add the equations vertically, and that unknown will cancel.*

We will then have one equation in one unknown, which we can solve.

Cases

Compatible systems:
(With solution)

- *Determined compatible system*
- *Undetermined compatible system*

Incompatible systems
(Without solution)

1. Solve for x and check:

$a)\ 2(3x-1)=2x+3$

$b)\ \frac{3x+1}{2}-\frac{x-4}{3}=2+x$

$c)\ \frac{2(x-1)}{3}-\frac{3(2x+2)}{4}-1=0$

$d)\ (x-3)^2+2x^2-x+5=3(x+1)^2$

2. Rearrange each of the formulae below to make *n* the subject:

$a)\ a=bn$ $b)\ r=2n-4$ $c)\ a=\frac{3n}{2}+1$ $d)\ y=\frac{3+b}{n}.$

3. Solve these questions:
   a. Find the sum and the product of the roots of $x^2+4x+3=0$
   b. Write a quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$
   c. Find the sum and the product of the roots in $4x^2+x-3=0$
   d. If one of the roots of $x^2-6x+k=0$ is 4, find the other root.
   e. If one of the roots of $x^2+9x+k=0$ is double than the other, find the roots.

4. Work out the discriminant and answer:
   a. Which of these equations has two real and different roots?
   b. Which has imaginary roots?
   c. Which has irratinal roots?
   d. Which has two real equal roots?
   e. Which has rational roots?

- $x^2-9=0$
- $x^2-2x+1=0$
- $x^2-x+1=0$
- $x^2-x-1=0$
- $x^2+14x+49=0$

5. John is twice as old as his friend Peter. Peter is 5 years older than Alice. In 5 years, John will be three times as old as Alice. How old is Peter now?

6. The length and breadth of a rectangle are (x + 4) cm and x cm respectively. Write the expression (i) the perimeter of the rectangle and (ii) the length of the side of a square with the same perimeter.If the sum of the areas of the square and the rectangle is 94cm^2,find x.

7. A small swimming pool can be filled by two pipes in 3 hours. If the larger pipe alone takes 8 hours less than the smaller pipe to fill the pool, find the time in which it will be filled by each pipe singly.

8. If 4 apples and 2 oranges equals $1 and 2 apples and 3 orange equals $0.70, how much does each apple and each orange cost?

9. Samantha has 30 coins, quarters and dimes, which total $5.70. How many of each does she have? *(1 quarter=0.25$, 1 dime=0.1$)*

10. Find the value of two numbers if their sum is 12 and their difference is 4.

11. The sum of the digits of a certain two-digit number is 7. Reversing its digits, increases the number by 9. What is the number?

12. The school that Stefan goes to is selling tickets to a choral performance. On the first day of tickets sales the school sold 3 senior tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior tickets and 2 child tickets. Find the price of a senior ticket And a child ticket.

13. How many pounds of coffee worth $1.44 a pound should be mixed with 20 pounds worth $2.80 a pound to produce a mixture worth $1.56 a pound?

NUMBER PATTERNS AND SEQUENCES

A ***sequence*** is a list of numbers that goes on according to a mathematical rule. Each value in the list is called a ***term***.
Examples of sequences:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

1, 4, 9, 16, 25, 36, 49, 64, 81, ...

***Rule* or formula for the *nth term*:** you can generate the terms of the sequence from their positions in the sequence. The letter *n* is normally used for the position, and a_n, b_n,.. etc for the nth term. E.g., in the sequence 1, 4, 9, 16, 25, 36, 49, 64, 81, ... $a_n = n^2$

Arithmetic Sequences

An Arithmetic Sequence is made by adding some value each time.

3, 8, 13, 18, 23, 28, 33, 38, ...

This sequence has a difference of 5 between each number. The pattern is continued by adding 5 to the last number each time.

Geometric Sequences

A Geometric Sequence is made by multiplying by some value each time

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number. The pattern is continued by multiplying the last number by 2 each time.

1. Find he missing terms in each sequence:
   a. 4, 7, 10, ..., 16,...
   b. 5, 6, 8, 11, ..., ...
   c. ...,..., 20, 40, 80, ...
   d. 10, 6, 2, ..., ...
   e. 83, ..., 74, 65, 53,....
   f. 5, 9, 14, 23, ..., ...
2. Write down the first four terms of these sequences:
   g. $a_n = 4n-2$
   h. $b_n = n^2+10$
   i. $c_n = 1/2n$
   j. $d_n = 2^n$ Each question gives the first five terms of a sequence. For each one:
   k. Calculate the 10^{th} term.
   l. Write down the nth term if ipossible.
      i. 6, 11, 16, 21, 26,...
      ii. 10, 9, 8, 7, 6,....
      iii. 0, 3, 8, 15, 24,....
      iv. 1, 1.2, 1.4, 1.6, 1.8,...
      v. 2, 4, 8, 16, 32,.....
      vi. 1/2, 3/4, 5/8, 7/16, 9/32,...

3. The sum of the interior angles of a triangle is 180º, of a quadrilateral is 360º and of a pentagon is 540º. Assuming this pattern continues, find the sum of the interior angles of a dodecagon (12 sides).

4. . After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

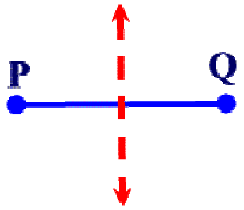
5. A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?

GEOMETRY

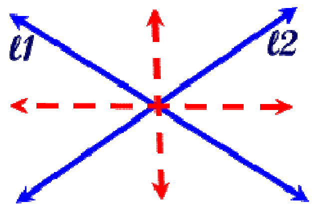
GEOMETRIC LOCI

Locus: is a **set of points** which satisfies a certain condition. (Locus is a Latin word meaning place; the plural of locus is loci). These are some examples of locus in the plane:

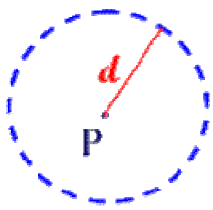
1. **Perpendicular bisector**: The locus of points equidistant from two points ***P*** and ***Q*** is the perpendicular bisector of the segment ***PQ***:



2. **Bisector:** The locus of points equidistant from two secant lines l_1 and l_2 is a pair of bisectors that bisect the angles formed by l_1 and l_2:



3. ***Circumference*:** The locus of points at a fixed distance ***d*** from one point ***P*** is a circumference with radius ***d*** and centre ***P:***

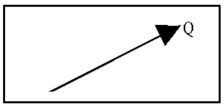


4. **Ellipse:** is the locus of points which sum of distances from two fixed points ***F*** and ***F'*** is the same, k.
5. **Hyperbola:** is the locus of points which difference of distances from two fixed points ***F*** and ***F'*** is the same, k.
   Elements of an ellipse and hyperbola: focuses, centre and axis
6. **Parabola:** is the locus of points whose distance from a fixed point ***F*** is the same as the distance from a fixed line ***d***. *The elements of the parabola are called focus, vertex and directrix*

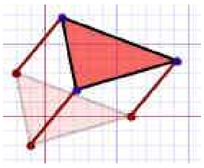
Movement is a geometric transformation that changes positions of shapes but not their sizes. Movements in the plane are translations, rotations, line symmetries (rotations), and point symmetries.

The original shape is called the ***object***. The result is the ***image*** shape. Images are congruent to their objects (they have the same shape and size).

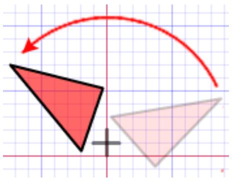
Vector : a vector is represented by an arrow, defining the direction, and the length of the arrow defines the vector's magnitude.



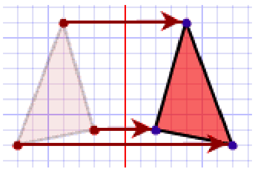
Translations: a translation moves a figure from one place to another. ***Vectors*** are used to describe the translation: in the vector u = (a, b), a represents the horizontal movement, and b represents the vertical movement.



Rotations: a rotation turns a figure through an ***angle*** about some fixed point. This fixed point is called the ***centre*** of rotation.

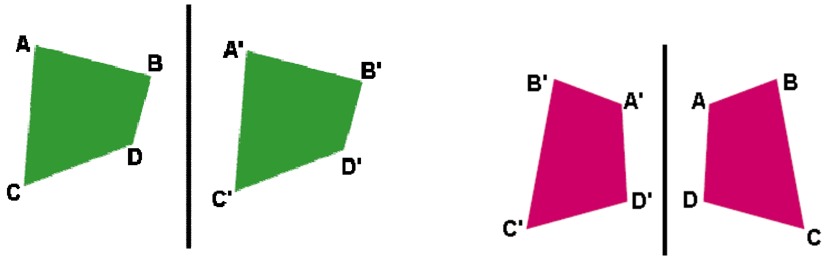


Reflexions or line symmetries: a reflexions creates an image of an object on the other side of the ***mirror line***. This line is called ***axis of reflexion***.



1. Answer these questions:

   a. Do the figures bellow illustrate line reflexions?



   b. A person is sitting 4 feet away from a mirrored wall. How far away from the person will his/her reflection appear to be?

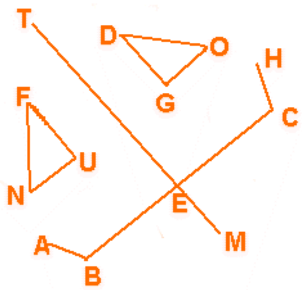
   c. In the diagram, line segment TM is the line of reflexion for the figure.

      i. What is the reflection image of segment *AB*?

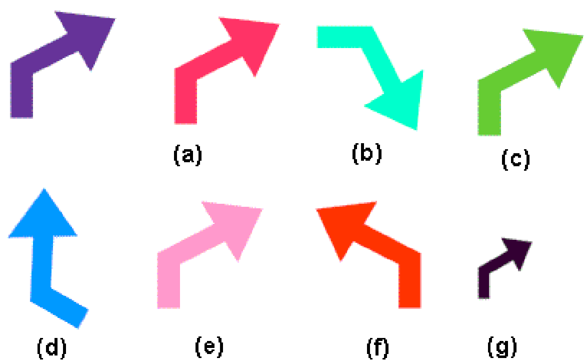
      ii. What is the reflection image of triangle *FUN*?

      iii. What is the reflection image of point *U*?

      iv. How does the length of segment *EC* compare to the length of segment *BC*?



   d. Which of the following lettered figures are translations of the shape of the purple arrow?

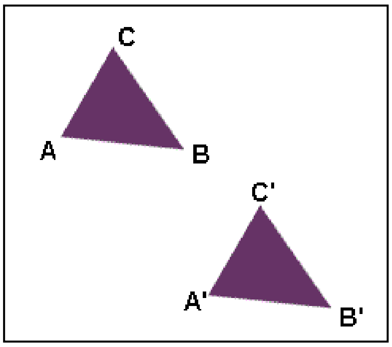


e. Which of the following translations best describes the diagram?

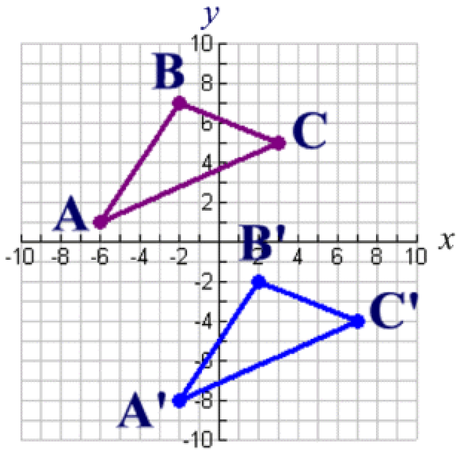
   i. 3 units right and 2 units down.

   ii. 3 units left and 2 units up

   iii. 3 units left and 2 units up



f. Which vector describes the translation seen on this set of axes?



g. Under the vector v= (-2, 3) the point (-3,-10) will be translated to the point.....

7. Draw a co-ordinate grid from -10 to 10, and draw the object: a triangle with vertices A (4, 0), B(4, 2) and C(3, 2). For each part, write down the co-ordinates of the vertices of the image:

   a. Trasnlate using these vectors:

      i. u = (0, 6)

      ii. v = (5, 3)

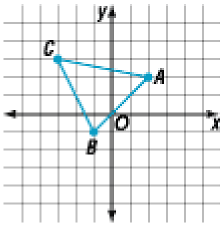
   b. Rotate as follows:

      i. 90º clockwise, centre (-3, 4)

      ii. 180º, centre (2, -2)

c. Reflect uin the following lines:
   i. y axis
   ii. x axis
   iii. origin (0, 0)

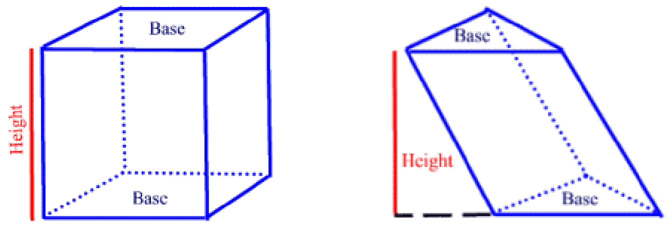
8. Solve these questions:
   a. Find the coordinates of A(7, 4) after it is reflected over the x-axis.
   b. Locate the image of B(1, -1) after a translation of v = (-2, 3)
   c. Graph the image of $\triangle ABC$ after a reflection across the *x*-axis



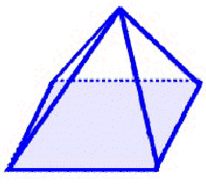
POLYHEDRA. PYTHAGOREAN THEOREM

A ***polyhedron*** is a three-dimensional solid whose faces are polygons joined at their edges. A polyhedron is said to be ***regular*** if its faces are made up of *regular polygons.* A regular polygon is a polygon with sides of equal length placed symmetrically around a common center.
Regular polyhedra:

Prisms are three-dimensional closed surfaces. A prism has two **parallel** faces, called bases, that are **congruent** polygons. The lateral faces are rectangles in a right prism, or parallelograms in an oblique prism. In a right prism, the joining edges and faces are perpendicular to the base faces.



Pyramids are three-dimensional closed surfaces. The **one base** of the pyramid is a polygon and the lateral faces are **always triangles** with a common vertex. The vertex of a pyramid (the point, or apex) is not in the same plane as the base.

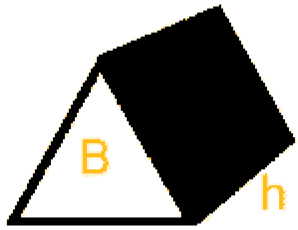


SURFACE AREAS AND VOLUMES

- The **volume** of a prism is the product of the base area times the height of the prism.

V = Bh

(Volume of a prism: *B* = base area, *h* = height)



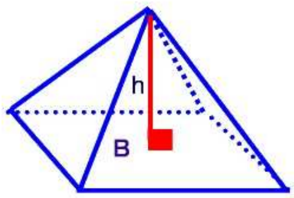
h = height(altitude) between bases
B = area of the base

- The **surface area** of a prism is the sum of the areas of the bases plus the areas of the lateral faces. This simply means the sum of the areas of all faces.

- The **volume** of a pyramid is one-third the product of the base area times the height of the pyramid.

V = 1/3 Bh

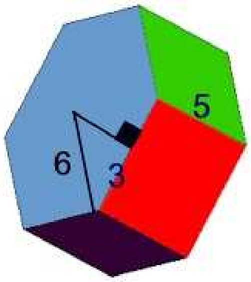
(Volume of a pyramid: *B* = base area, *h* = height)



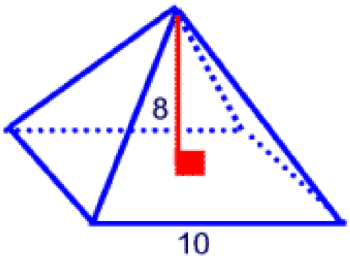
h = height (altitude) from vertex to base
B = area of base

- The **surface area** of a pyramid is the sum of the area of the base plus the areas of the lateral faces.
This simply means the sum of the areas of all faces

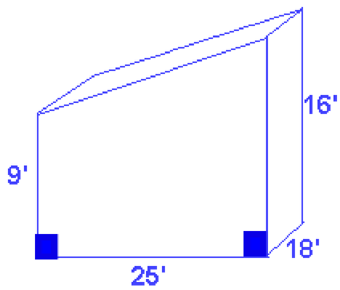
1. This figure is a regular hexagonal prism. Find its volume:



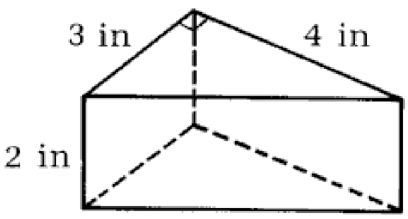
2. A regular pyramid is shown below. Find the volume of the pyramid to the *nearest cubic unit.* Find the lateral area too.



3. Find the volume in cubic feet.



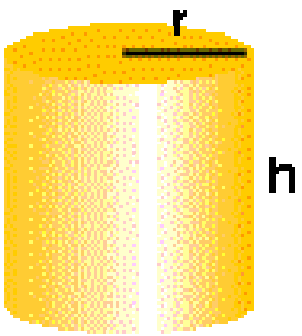
4. This figure represents a slab *(bloque, cuña)* of cheese. It is in the form of a right triangular prism. Find the least amount of wrapping *(papel de envolver)* needed to cover the cheese on all sides:



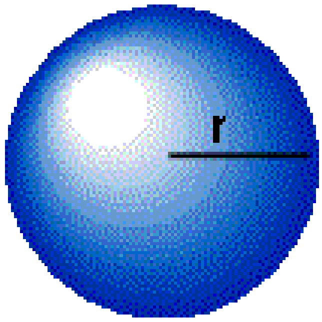
5. Work out the volume of a cuboid 17 cm long, 4 cm wide and 15 cm high.

6. A pool is 3 m wide, 12 m lond and 1 m deep. What is its volume?

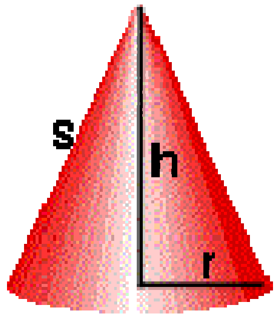
SOLIDS OF REVOLUTION



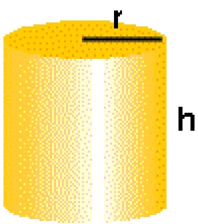
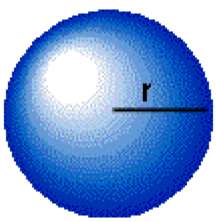
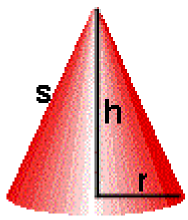
h = height (altitude)
r = radius



r = radius



h = height (altitude)
r = radius
s = slant height

	**Cylinder** $V = \pi r^2 h$ $SA = 2\pi r h + 2\pi r^2$
	**Sphere** $V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2 = \pi d^2$
	**Cone** $V = \frac{1}{3}\pi r^2 h$ $SA = s\pi r + \pi r^2$

7. Soda is sold in aluminum cans that measure 6 inches in height and 2 inches in diameter. How many cubic inches of soda are contained in a full can?
(*Round answer to the nearest tenth of a cubic inch.)*

8. A pharmacist is filling medicine capsules. The capsules are cylinders with half spheres on each end. If the length of the cylinder is 12 mm and the radius is 2 mm, how many cubic mm of medication can one capsule hold? (*Round answer to the nearest tenth of a cubic mm.)*

9. Find the volume of a cone with height of 9 cm and radius of 7 cm.

10. What is the height of a cone with a volume of 132 cubic cm and radius of 3 cm?

11. Are these statements true or false?

a. A cone is a polyhedron

b. A prism has 4 bases.

c. The volume of a cube with edges of 5 cm is 125 cubic cm.

d. The faces of a polyhedron are all flat.

e. The volume of a sphere with radius of 1 ft is about 4.19 cubic ft.

f. The formula for the vomue of a pyramid is ½ Bh

12. Suppose a water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. How much sheet metal was used in its construction?

13. Answer these questions:

a. What is the volume of Earth? What is ist surface area?
$R_{Earth} = 6 \cdot 10^6 m.$

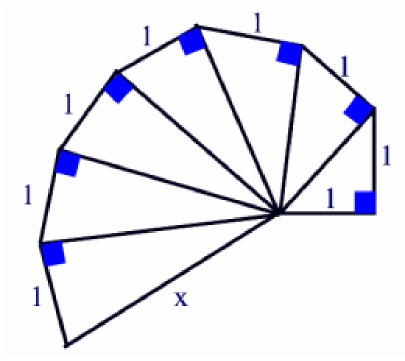
b. What is the volume of Moon? What is ist surface area? $R_{Moon} = 1 \cdot 10^6 m.$

c. Find the ratio of radii of Earth and Moon.

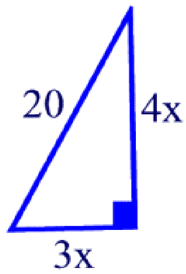
d. Find the ratio of their volumes.

e. Find the ratio of their surface areas.

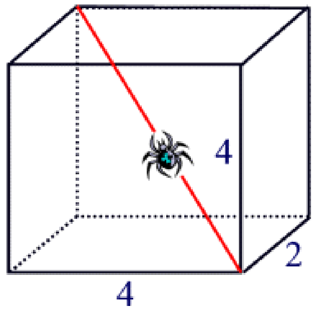
14. Find the length of segment x in the figure:



15. Find x in this triangle:

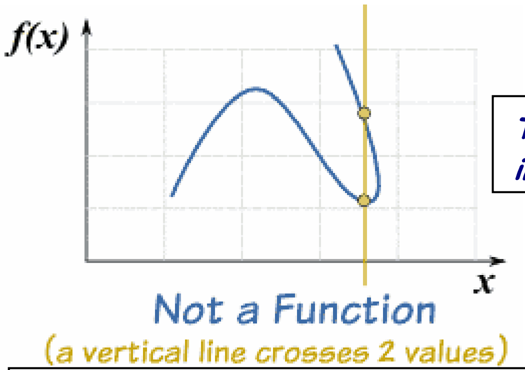


16. A spider has taken up residence in a small cardboard box which measures 2 inches by 4 inches by 4 inches. What is the length, in inches, of a straight spider web that will carry the spider from the lower right front corner of the box to the upper left back corner of the box?



FUNCTIONS

Function: A function is a set of ordered pairs in which each *x*-element **(independent variable)** has only ONE *y*-element associated with it.

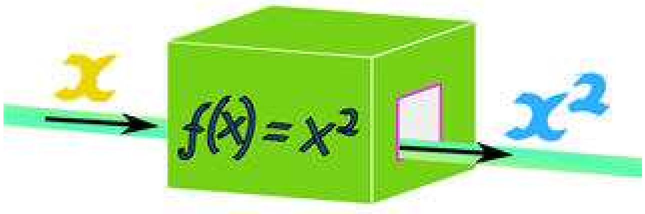


The graph shows that a vertical line may intersect more than one point in a relation

If the graph shows that a vertical line intersects only ONE point, this is a function. This is called the vertical line test for functions.

Functions can be defined by a chart (table), a graph or an equation or formula.

$f(x) = x^2$ shows you that function "***f***" takes "***x***" and squares *it*.



All these notations are equivalent:

$y = x^2$

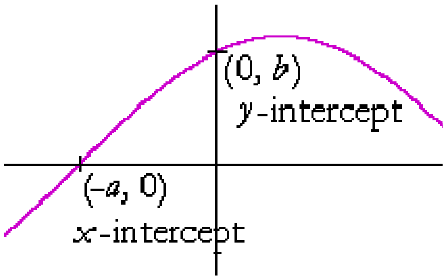
$f(x) = x^2$

$x \rightarrow x^2$

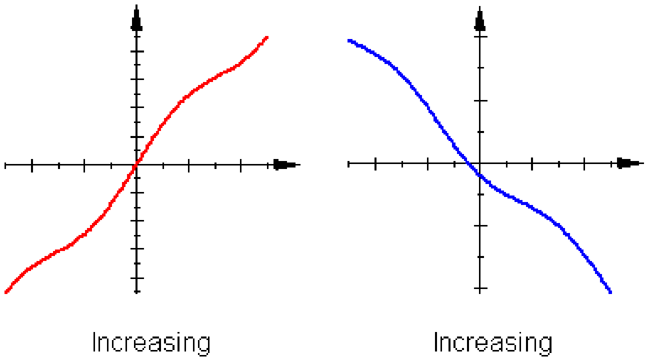
PROPERTIES OF FUNCTIONS

- **Domain**: is the set of all the x-coordinates of a function

- *x*-intercept and *y*-intercept of a graph



- **Increasing** and **decreasing** functions



- **Extremes** of a function: *maximum* and *minimum*

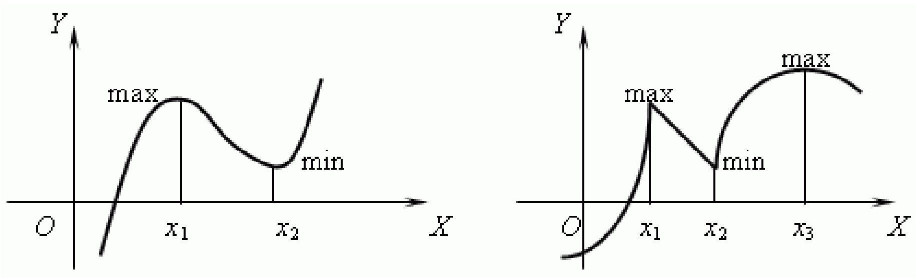
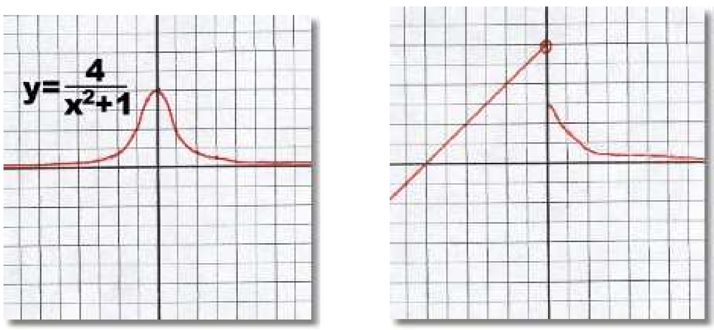


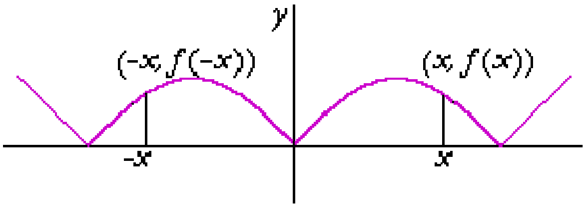
Fig. 5*a*

Fig. 5*b*

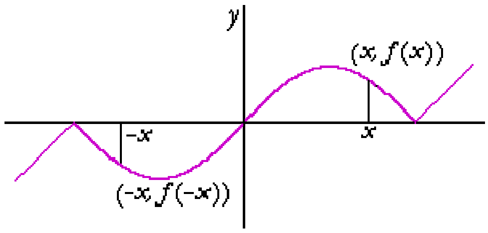
- **Continuous** and **discontinuous** functions



- **Symmetric** functions

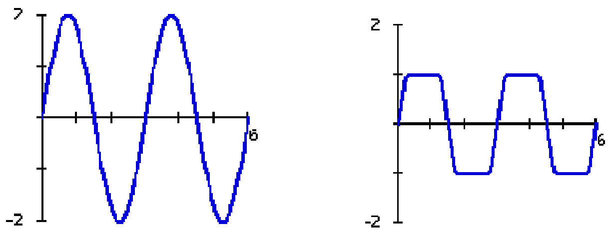


Even function



Odd function

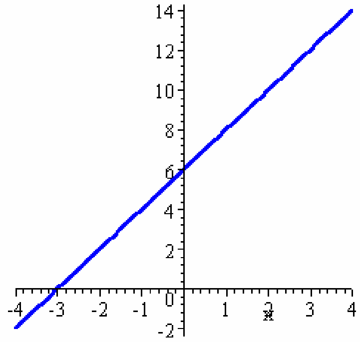
- **Periodic** functions



LINEAR FUNCTIONS

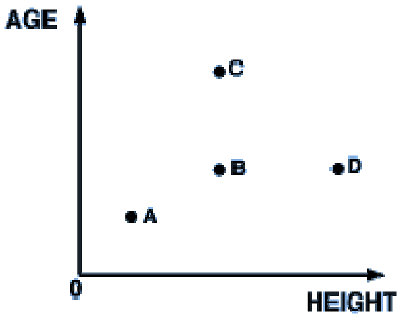
Linear functions are functions with an equation like this one: ***y = ax + b.*** Their graphs are straight lines.

- The **slope** (also known as gradient) of a linear function is ***a,*** and the y-intercept is ***b.***
- To calculate the gradient of a line on a graph, select two points on it and:
  - Work out the increase in x from one point to the other and the increase in y.
  - The quotient (increase in y)/(increase in x) gives the gradient
- Example: the line $y = 2x + 6$ has slope a = 2 and y-intercept b = 6. Its graph is:



EXERCISES

1. Match the names of the people to the letters on the graph below:.

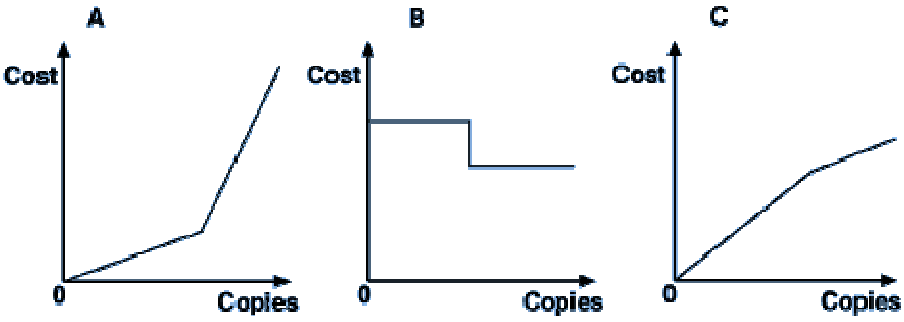


- *Sophie is the tallest.*
- *George and Waseem are the same height.*
- *Freda is the shortest.*
- *George is the oldest.*
- *Sophie and Waseem are the same age.*
- *Freda is the youngest*

2. A company uses the following charges for photocopying:

50 copies or less - 20p each
Extra copies - 10p each

Which of the following graphs A, B or C could show how the cost changes with the number of copies?



3. For the following question look at the two variables and decide which should be on the horizontal and which should be on the vertical axis if you were to draw a graph: a) time and temperature, b) time and distance, c) distance travelled and number of rest stops.

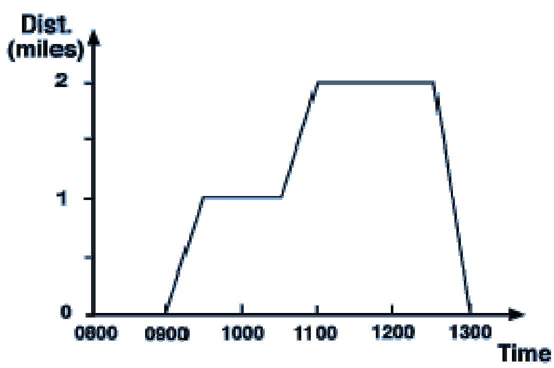
4. Video Star is a rental company which hires out videos. The cost of hiring is £2 per day.
    a. *Complete the table below to show the hire costs for different numbers of days*
    b. *Using squared paper draw a graph to show the cost of hiring the videos from Video Star.*

Number of days	1	2	3	4	5	6	7	8	9	10
Cost in £	2									

5. The Brown family are visiting Scotland and decide to hire a car for a week. There are two rival companies which offer different schemes. They are Allcars who charge £210 per week all inclusive and Bighires who charge £150 plus 10 pence per mile.
    a. *Complete the table below to show the hire costs for different distances.*
    b. *Draw a graph to show the hire costs using the Allcars scheme and, on the same grid, draw a graph showing the Bighires costs.*
    c. *The Browns are planning to drive about 100 miles every day. Which scheme would be better for them?*

Number of miles	0	200	400	600	800	1000
"Allcars" cost in £						
"Bighires" cost in £						

6. The graph below shows how Jan's distance from home varied with time as she walked to her friend's house, bought a magazine in a shop on the way, and walked back home again.



a. How far is it from Jan's house to the shop?
b. How far is it from Jan's house to her friend's?
c. How long did Jan spend in the shop?
d. How long did Jan spend at her friend's?
e. How long did the trip take altogether?

7. Amy has a mobile phone with this tariff: *Monthly charge £10, calls cost 10 p per minute*.
   a. Complete the table which shows Amy's costs:

Number of minutes							
Cost £							

   b. Draw the graph of the function " Number of minutes-Cost "
   c. How much would 35 minutes of calls cost?
   d. If Amy pays £25, for how many minutes did she use the phone that month?
   e. Write the equation of this function.

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